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Q Degradation in Varactor-Tuned Oscillators

S. F. PAIK, MEMBER, IEEE

***Abstract*—A general analysis of varactor-tuned negative-resistance oscillators is presented to show the varactor loading effect on the oscillator Q. The external Q of the varactor loaded circuit normalized with respect to the unperturbed external Q is plotted as a function of the tuning range for several values of the varactor Q and the capacitance ratio.**

In varactor-tuned oscillators, the primary factor limiting the tuning range is the resistive loading effect of the varactor diode. As the diode is coupled strongly into the oscillator circuit, the tuning range increases at the expense of the oscillator Q. The tuning range attainable with a given diode, therefore, is determined by the allowable degradation of the oscillator Q and the accompanying power dissipation. Cawsey [1] has presented an analytical evaluation of this effect, but his results did not include an explicit relation between the oscillator Q and the tuning range. Inasmuch as several critical parameters of oscillators, such as the FM noise and the temperature stability, are directly related to the external Q of the oscillator, it is desirable to have a quantitative measure of the tradeoff between the oscillator Q and the tuning range. In this note, we present an analysis of varactor-tuned negative-resistance oscillators and derive a general expression for the external Q as a function of the commonly quoted varactor parameters and the tuning range.

Fig. 1 shows circuit models of a varactor-tuned negative-resistance oscillator (e.g., IMPATT or Gunn diode oscillators). In practice, both the varactor and the load may be coupled to the negative-resistance element through a transformer, and the circuit parameters indicated in Fig. 1 refer to transformed quantities. Following the common practice, the varactor diode is represented by a series RC circuit, in Fig. 1(a), and all parasitic elements are assumed to be included in the main resonator. This equivalent circuit is valid at frequencies near resonance if the varactor impedance (including the package parasitics) is a smooth function of frequency and bias voltage devoid of loops in the impedance plot.

The varactor capacitance C_V is a function of the bias voltage, decreasing monotonically from C_{V0} at zero bias to C_{VB} at breakdown. The series resistance R_S is assumed to be constant at all bias levels. Varactors are usually characterized by two invariant parameters defined in terms of C_V and R_S: r, the capacitance ratio, and its quality factor Q_V:

$$r = \frac{C_{V0}}{C_{VB}} > 1 \tag{1}$$

$$Q_V = \frac{1}{(\omega R_S C_{V0})}. \tag{2}$$

Since the oscillator circuit in Fig. 1 is represented by a shunt resonant circuit, it is convenient to convert the varactor into its shunt equivalent form as shown in Fig. 1(b). The shunt elements are

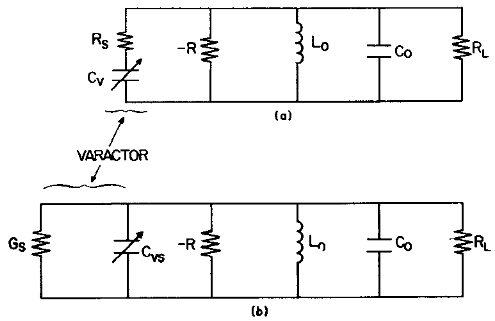

Fig. 1. (a) Equivalent circuit of a varactor-tuned negative-resistance oscillator with the varactor represented by a series RC circuit. (b) With the varactor represented by a parallel RC circuit.

$$G_S = \left(\frac{1}{R_S}\right)\left[1 + Q_V^2\left(\frac{C_{V0}}{C_V}\right)^2\right]^{-1} \simeq \left[R_S Q_V^2 \left(\frac{C_{V0}}{C_V}\right)^2\right]^{-1} \tag{3}$$

$$C_{VS} = C_V[1 + (C_V/C_{V0})^2/Q_V^2]^{-1} \simeq C_V. \tag{4}$$

The approximation is valid if $Q_V^2 \gg 1$.

In the absence of the varactor, the resonant frequency and the external Q of the oscillator are

$$\omega_0 = (L_0 C_0)^{-(1/2)} \tag{5}$$

$$Q_{ext} = \omega C_0 R \tag{6}$$

where

$$R = |-R| = R_L.$$

When the varactor is introduced, the frequency of the oscillation can be tuned from

$$\omega_1 = [L_0(C_0 + C_{V0})]^{-(1/2)}$$

to

$$\omega_2 = [L_0(C_0 + C_{VB})]^{-(1/2)}.$$

Assuming the difference between C_{V0} and C_{VB} is a small fraction of the total capacitance $(C_0 + C_{V0})$, the fractional tuning range referenced to ω_1 may be expressed as a function of the capacitance ratio as follows:

$$\Delta f/f_1 \simeq (1/2)[(C_{V0} - C_{VB})/(C_0 + C_{V0})] = \frac{1}{2}\frac{(1 - 1/r)}{(1 + C_0/C_{V0})}. \tag{7}$$

The external Q of the varactor-tuned oscillator reaches its lowest value at zero bias, since the varactor at zero bias introduces the greatest perturbation and its own Q is at its lowest. The external Q of the oscillator at zero bias is

$$Q'_{ext} = \omega(C_0 + C_{V0})\left[\frac{RR_SQ_V^2}{R + R_SQ_V^2}\right] = \frac{Q_{ext}Q_V^2}{(R/R_S) + Q_V^2} + \frac{Q_V}{1 + (R_S/R)Q_V^2}. \tag{8}$$

This expression may be normalized with respect to the unperturbed external Q in terms of the capacitance ratio, the tuning range, and the normalized varactor Q using (2), (6), and (7):

$$\frac{Q'_{ext}}{Q_{ext}} = [1 + 2xr(q - 1)/(r - 1)]^{-1} \tag{9}$$

where

$$x = \Delta f/f_1$$

and

$$q = Q_{ext}/Q_V.$$

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The author is with the Special Microwave Devices Operation, Raytheon Company, Waltham, Mass. 02154.

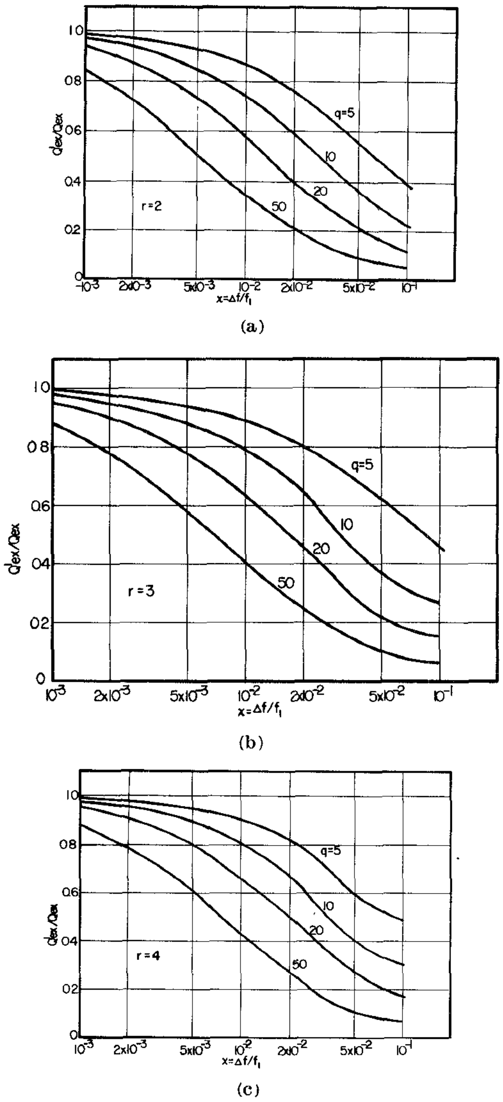

Fig. 2. Normalized external Q of the oscillator versus tuning range. (a) For $r = 2$. (b) For $r = 3$. (c) For $r = 4$.

Equation (9) is a general relation that should be valid for the oscillator circuit reduced to the general form shown in Fig. 1 (or its dual). The maximum tuning range for its validity is limited to less than 10 percent. It may be used for quantitative assessments of the varactor loading effect, or for the choice of varactor parameters to satisfy given design objectives. Alternately, this relation may be used as a theoretical basis for a new method of measuring varactor Q's. Equation (9) is plotted as a function of x in Fig. 2 for various values of q and r.

The effect of the varactor loading on the oscillator power output cannot be evaluated correctly without the knowledge of the nonlinear characteristics of the negative-resistance element. If the perturbation introduced by the varactor resistance is small (i.e., $R_S Q_V^2 \gg R$), however, the power output reduction can be estimated simply from the power dissipated in the varactor. The power output normalized with respect to the unperturbed output is

$$P_0'/P_0 = \left[1 + \frac{R}{R_S Q_V^2}\right]^{-1}. \tag{10}$$

Using (2), (6), and (7), this equation may be reduced to

$$P_0'/P_0 = \left[1 + \frac{2xrq}{(r-1) - 2xr}\right]^{-1}. \tag{11}$$

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